STATICALLY INDETERMINATE STRUCTURES

- 1 Statically indeterminate structures
  - Introduction, systematic approach
  - Statically indeterminancy
  - Examples
- 2 Braced Structures (non moving nodes)
  - Choice of statically determinate principal system and the systematic choice of the unknowns
  - Examples
- 3 Changes in Stiffness
- 4 Special situations
  - Support settlements
  - flexible joints

Last meeting if time is left over

- 5 Unbraced Structures
  - Introduction and systematic approach
  - Unknowns using the mixed or hybride solution technique
  - Examples
- 6 Introduction to the Displacement Method
  - Introduction and system

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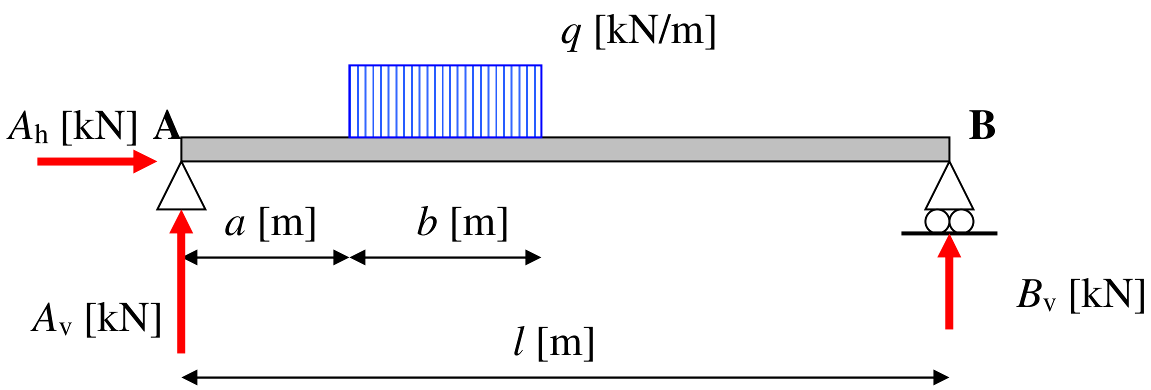
STATICALLY DETERMINATE STRUCTURES

DEFINITION:

FORCE DISTRIBUTION CAN BE DETERMINED BASED UP ON EQUILIBRIUM **SOLELY**.



3 unknown support reactions
3 equilibrium conditions

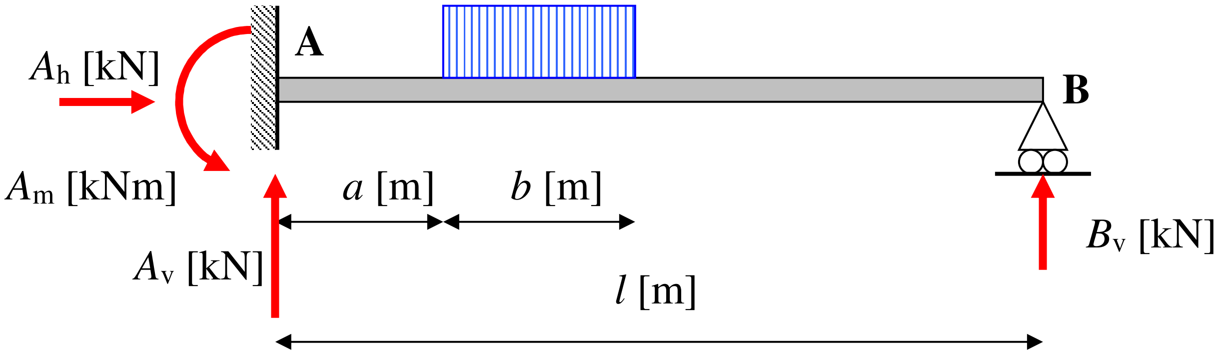
$$\sum F_h(\rightarrow) = 0 \quad \sum F_v(\uparrow) = 0$$
$$\sum T = 0$$

EXTERNAL EQUILIBIRUM

STATICALLY INDETERMINATE STRUCTURES

DEFINITION:

FORCE DISTRIBUTION CAN NOT SOLELY BE FOUND BASED ON EQUILIBRIUM, ADDITIONAL INFORMATION IS REQUIRED.



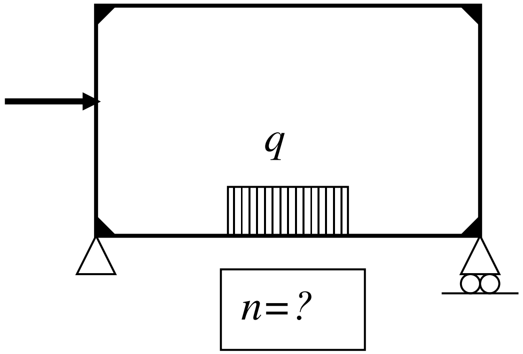
4 unknown support reactions
3 equilibrium conditions

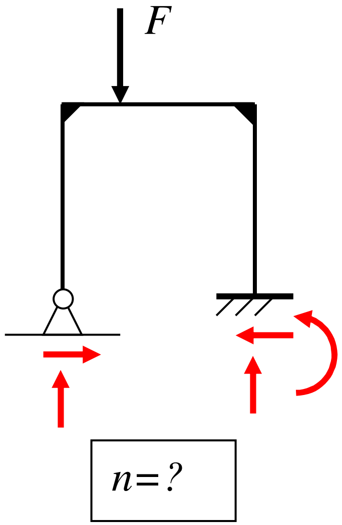
$$\sum F_h(\rightarrow) = 0 \quad \sum T = 0$$

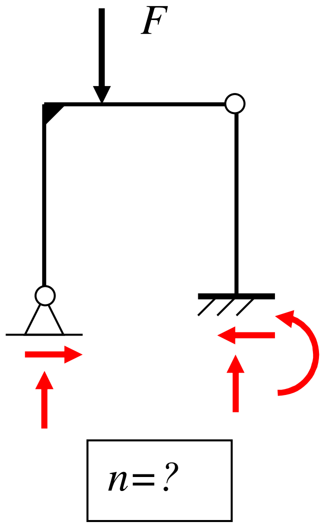
$$\sum F_v(\uparrow) = 0$$

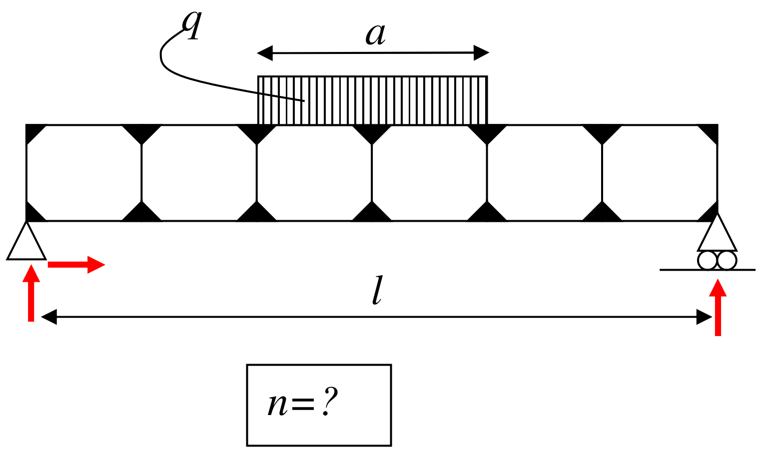
(simply) STATICALLY INDETERMINATE $n>0$

EXAMPLES

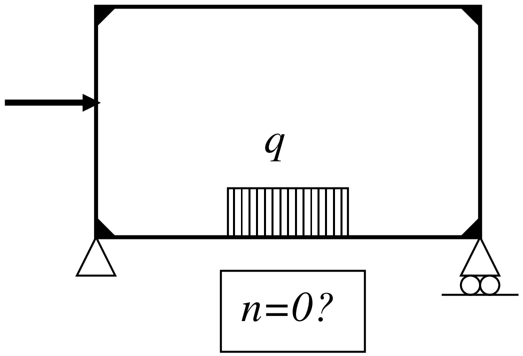


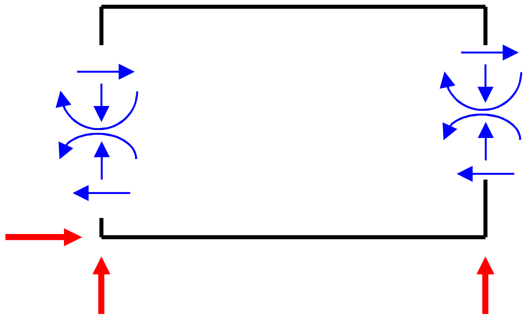






ANSWERS

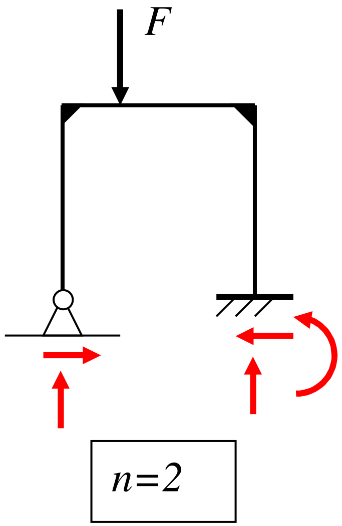


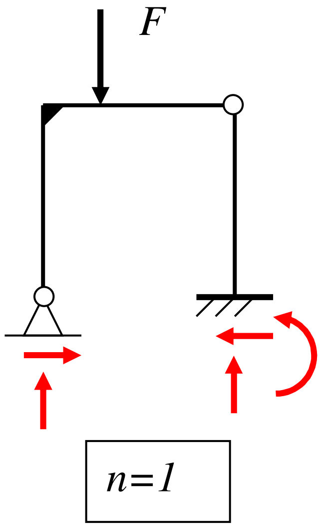


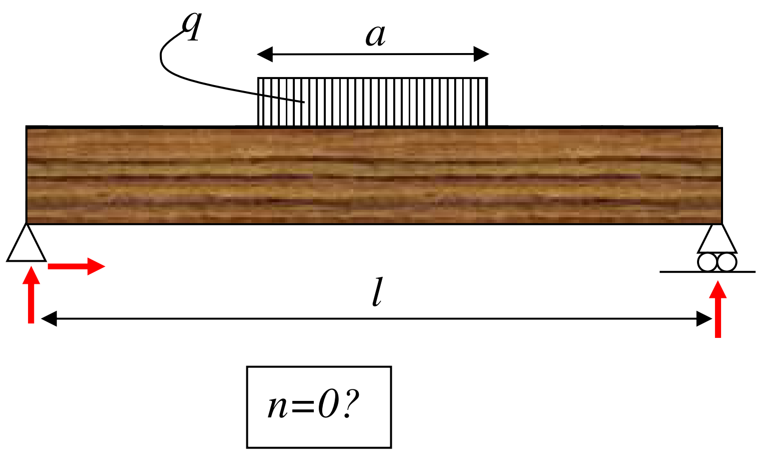
$$n = r + if - e$$

$$n = 3 + 6 - 2\times 3 = 3$$

n	= degree of statically indeterminancy
r	= nr of unknown support **r**eactions
if	= nr of **i**nteraction **f**orces
e	= nr of **e**quilibrium conditions







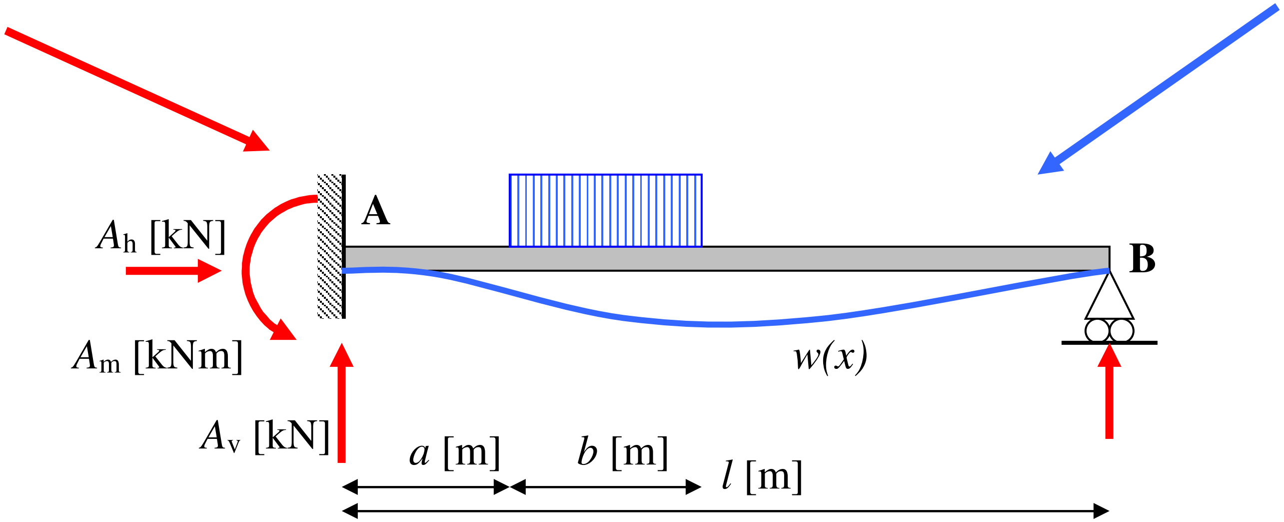
FORCE DISTRIBUTION IN STATICALLY INDETERMINATE STRUCTURES

FORCE METHOD

Fundamental unknowns such as support reactions or member forces

DISPLACEMENT METHOD

Fundamental unknowns are displacements (or rotations) of the nodes or members.



FORCE

METHOD

- Suitable for hand calculations
- Minimize work by “smart” engineering choices
- Practise and understanding required

DISPLACEMENT METHOD
(stiffness or matrix method)

- Systematic method
- Most suitable to be programmed
- Engine for FEM computerprograms

Kinematic relations

$$\kappa = -\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Constitutive relations

$$M = EI\kappa = -EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Equilibrium conditions

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V \quad \text{en} \quad \frac{\mathrm{dV}}{\mathrm{d}x} = -q(x)$$

MIXED or HYBRID METHOD

- Both forces and displacements are unknown

FORCE METHOD

Do not only consider **EQUILIBRIUM** but also the **DEFORMATIONS** of the structure due to the loading.

APPROACH:

NUMBER OF EQUILIBRIUM CONDITIONS

+

NUMBER OF DEFORMATION CONDITIONS

=

NUMBER OF UNKNOWNS

SYSTEMATIC APPROACH for the FORCE METHOD

- Choose a statically determinate **PRINCIPAL SYSTEM**
- Mark the **STATICALLY UNKNOWN(S)**
- Set up the corresponding **DEFORMATION CONDITION(S)**
- Elaborate the **DEFORMATION CONDITION(S)** with either the **REDUCED MOMENT AREA THEOREME** or the **FORGET-ME-NOTS**
- Solve the **STATICALLY UNKNOWN(S)** from these conditions
- Determine the **LOAD DISTRIBUTION** (*M*-, *V*- and *N*-lines)

RELATION BETWEEN STATICALLY UNKNOWNS AND THE CORRESPONDING DEFORMATION CONDITIONS

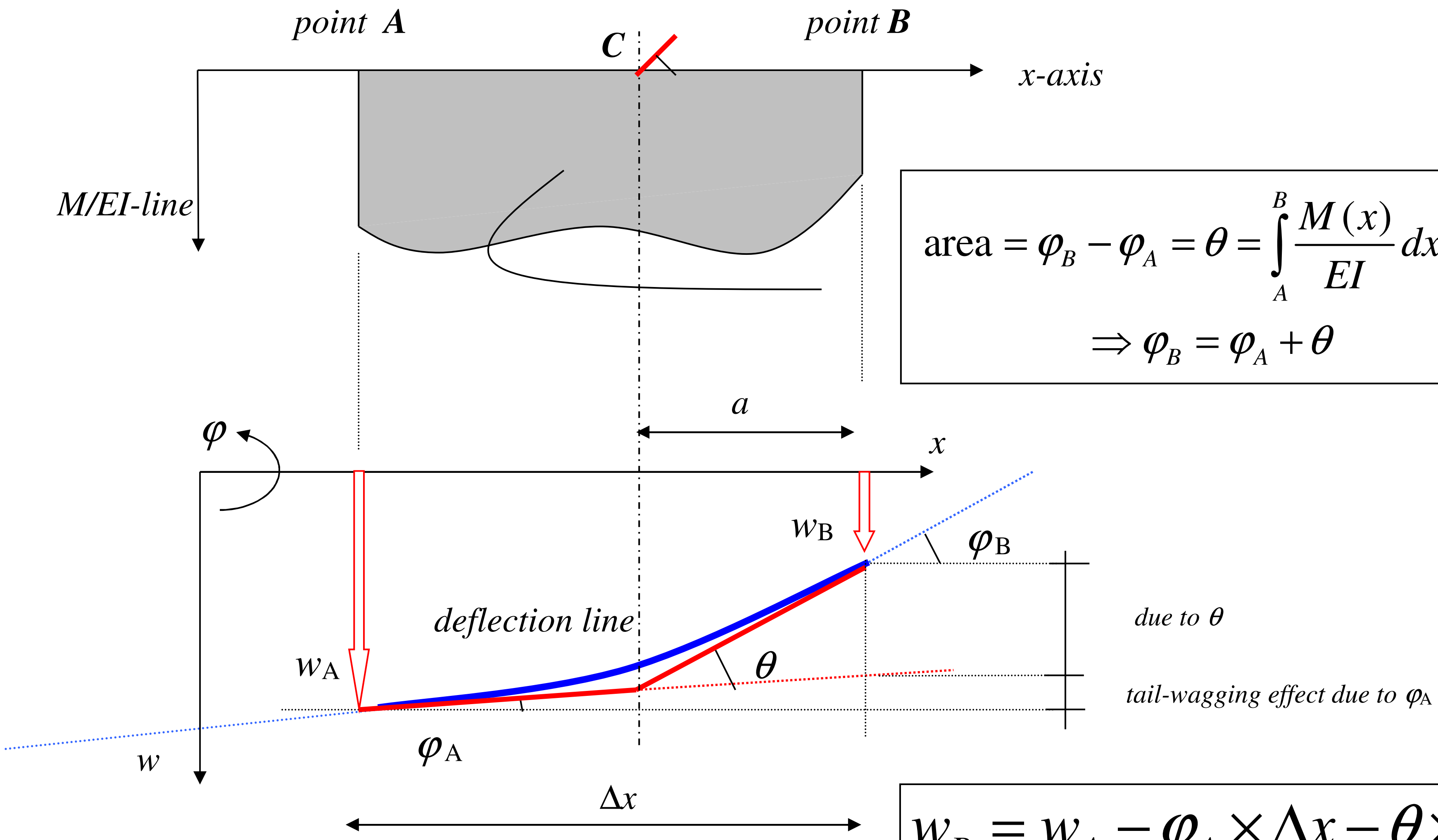
S.U is a FORCE

→ Deformation condition based up on a displacement

S.U. is a MOMENT

→ Deformation condition based upon a rotation

REDUCED MOMENTAREA THEOREME

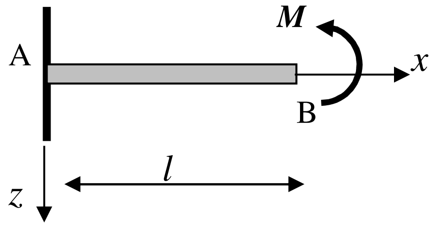


$$\text{area} = \varphi_B - \varphi_A = \theta = \int_A^B \frac{M(x)}{EI}dx = \int_A^B \kappa(x)dx$$

$$\Rightarrow \varphi_B = \varphi_A + \theta$$

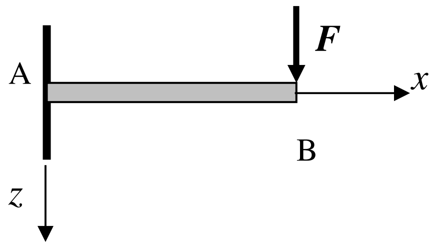
$$w_B = w_A - \varphi_A \times \Delta x - \theta \times a$$

FORGET-ME-NOTS : ENGINEERING FORMULAS



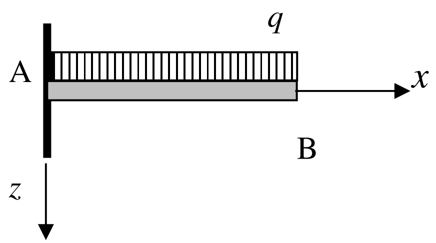
$$\varphi_B = \frac{Ml}{EI}$$

$$w_B = -\frac{Ml^2}{2EI}$$



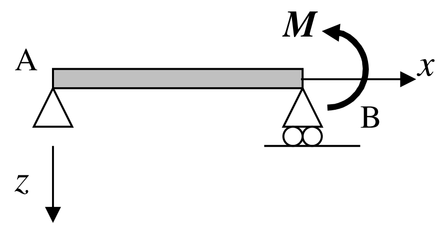
$$\varphi_B = -\frac{Fl^2}{2EI}$$

$$w_B = \frac{Fl^3}{3EI}$$



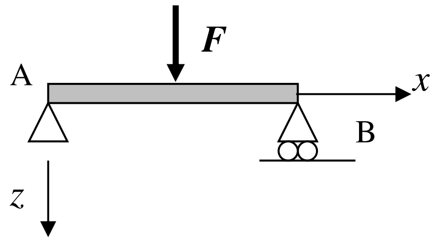
$$\varphi_B = -\frac{ql^3}{6EI}$$

$$w_B = \frac{ql^4}{8EI}$$



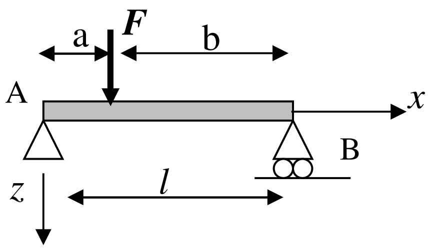
$$\varphi_A = -\frac{Ml}{6EI}$$

$$\varphi_B = \frac{Ml}{3EI} \qquad w_{midden} = \frac{Ml^2}{16EI}$$



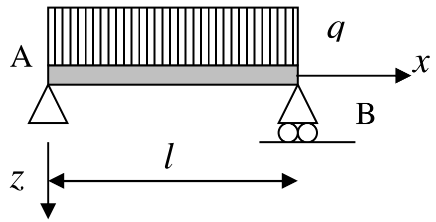
$$\varphi_A = -\frac{Fl^2}{16EI} \qquad \varphi_B = \frac{Fl^2}{16EI}$$

$$w_{\max} = \frac{Fl^3}{48EI}$$



$$\varphi_A = -\frac{Fab(l+b)}{6EIl}$$

$$\varphi_B = \frac{Fab(l+a)}{6EIl}$$

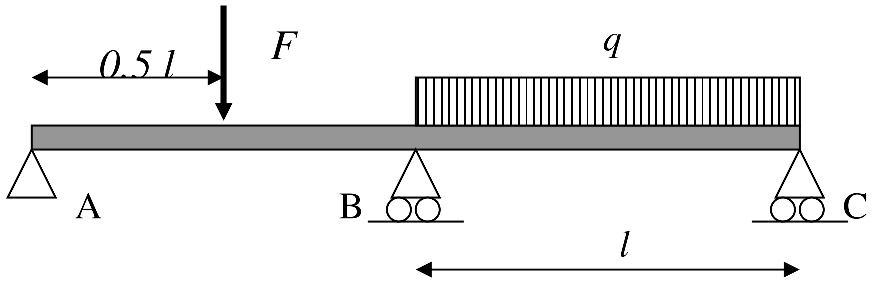


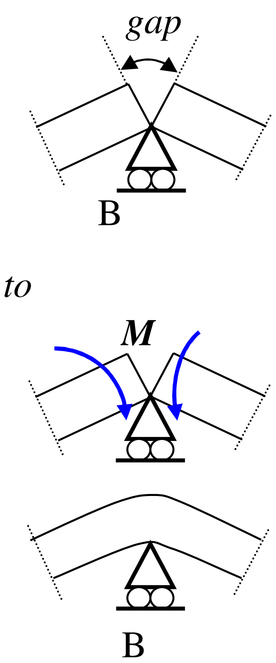
$$\varphi_A = -\frac{ql^3}{24EI} \qquad \varphi_B = \frac{ql^3}{24EI}$$

$$w_{\max} = \frac{5ql^4}{384EI}$$

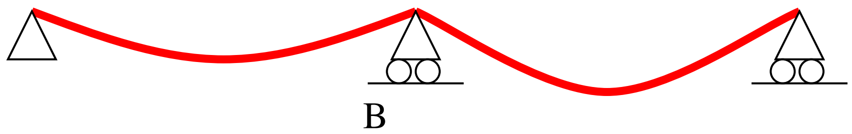
PROBLEM DESCRIPTION FOR STATICALLY INDETERMINATE STRUCTURES (BRACED)

DEGREE of
INDETERMINANCY:
***n*=1**



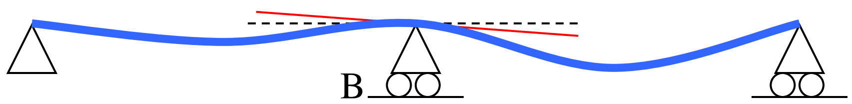


required moment to close the gap



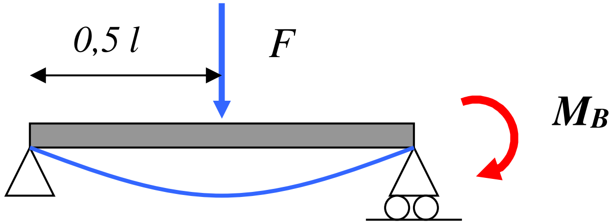
Two separate beams, both statically determinate by splitting the structure at the mid support.

➔ ***Dent in deflection***

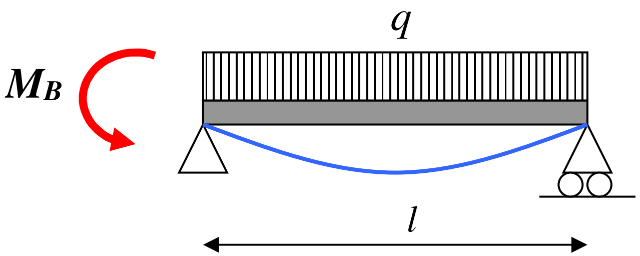


actual deformation without a dent at B
rotation to the left = rotation to the right

ELABORATE THE CONCEPT ...



1 static unknown (S.U.), M_B

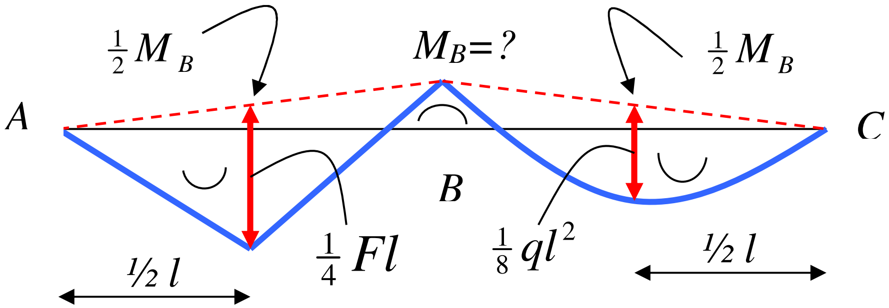


1 deformation or deflection condition

$$\varphi_B^{(AB)} = \varphi_B^{(BC)}$$

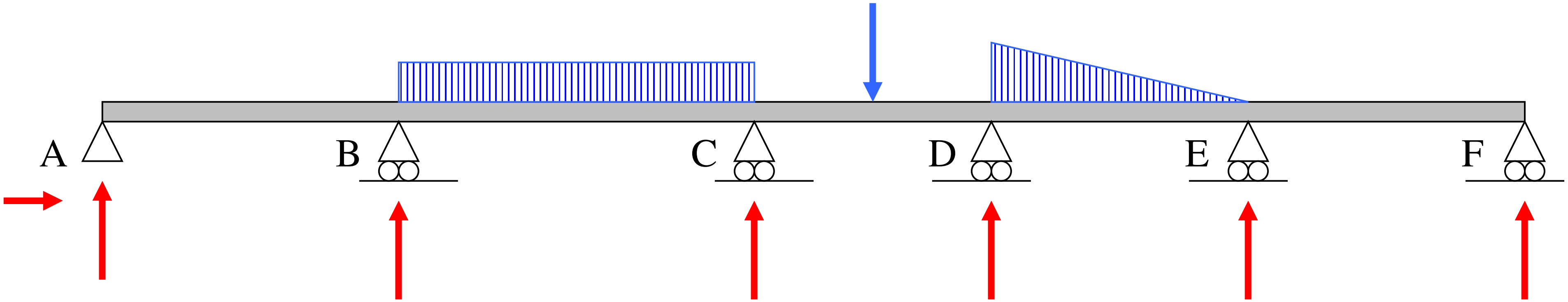
In this form also expressed as:

- **rotational condition**
- gap condition



Solve the S.U. with the forget-me-nots

CONTINUOUS BEAMS



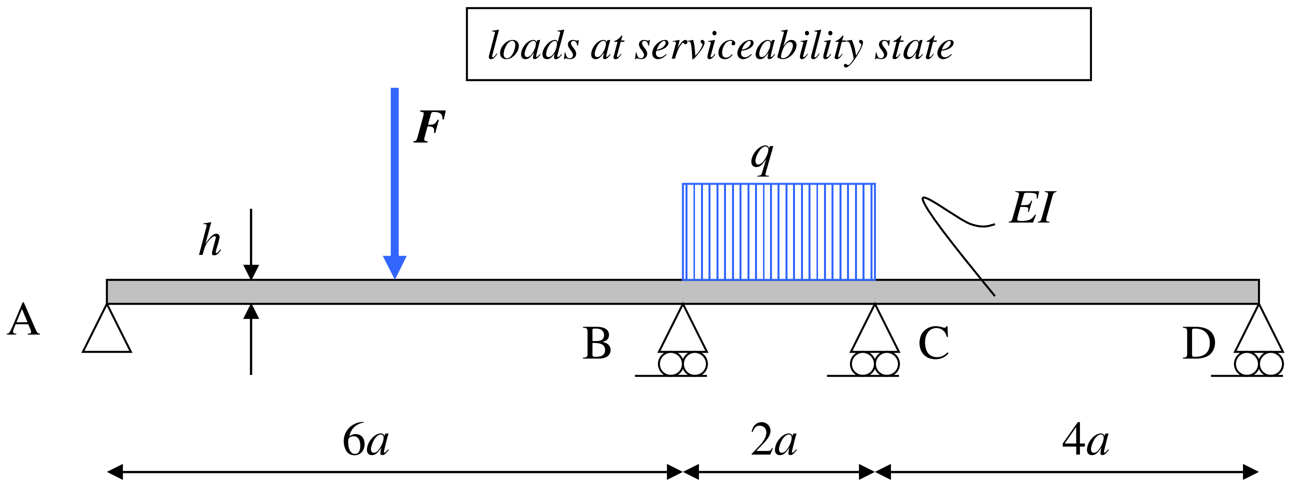
Unknown support reactions : 7
Number of equilibrium conditions : 3

DEGREE of STATIC INDETERMINANY : n= 4

THUS : **4** Static Indeterminants **AND**
4 Deformation conditions **!**

$$\varphi_B^{(BA)} = \varphi_B^{(BC)}$$
$$\varphi_C^{(CB)} = \varphi_C^{(CD)}$$
$$\varphi_D^{(DC)} = \varphi_D^{(DE)}$$
$$\varphi_E^{(ED)} = \varphi_E^{(EF)}$$

EXAMPLE 1 : CONTINUOUS BEAM (*design stage*)



Variable data :

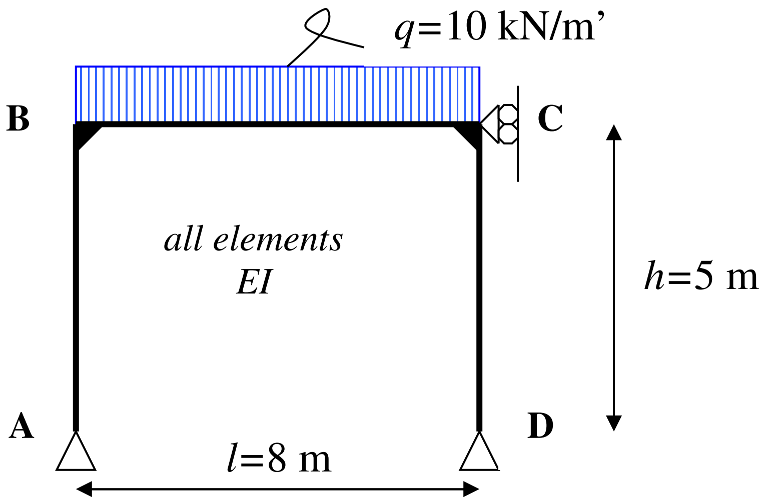
F = 100 kN
q = 10 kN/m
a = 1,0 m
h = 0,26 m

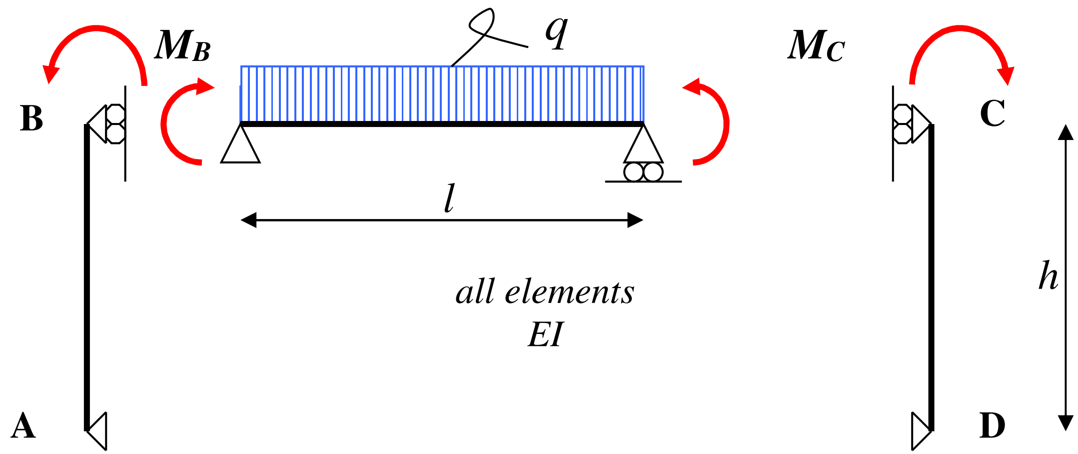
Fixed data :
EI = 1000 kNm^2
S235

Questions :

- Determine the force distribution
- Determine the deflection at the force *F*
- **conclusion ?**

EXAMPLE 2 : FRAME (BRACED)





$$\varphi_B^{(BA)} = \varphi_B^{(BC)}$$

$$\varphi_C^{(CB)} = \varphi_C^{(CD)}$$

FORGET-ME-NOTS

$$\frac{M_B h}{3EI} = -\frac{M_B l}{3EI} - \frac{ql^3}{24EI} - \frac{M_C l}{6EI}$$

$$\frac{M_B l}{6EI} + \frac{ql^3}{24EI} + \frac{M_C l}{3EI} = -\frac{M_C h}{3EI}$$

SOLUTION

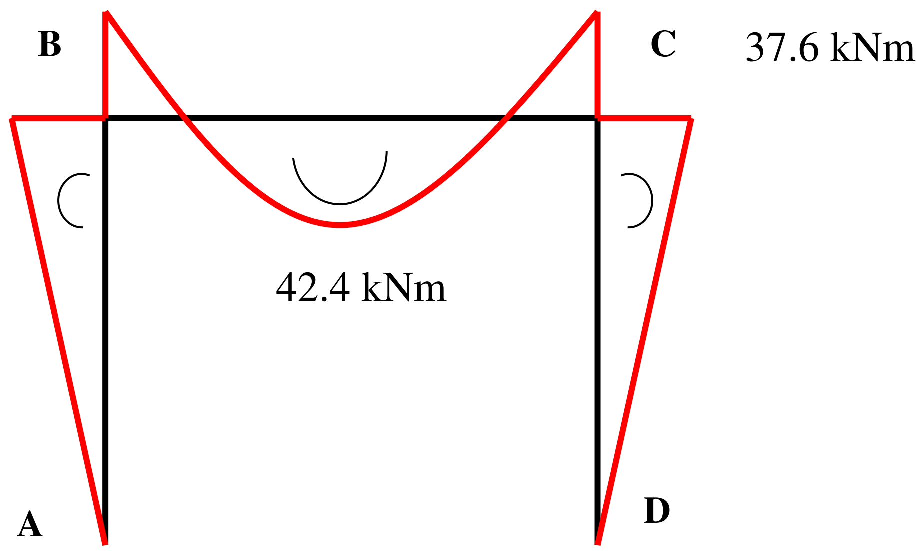
$$\frac{13M_B}{3}+\frac{8M_C}{6}=-\frac{10\times 8.0^3}{24}$$

$$\frac{8M_B}{6}+\frac{13M_C}{3}=-\frac{10\times 8.0^3}{24}$$

$$M_B=-37.6\ \text{kNm}$$

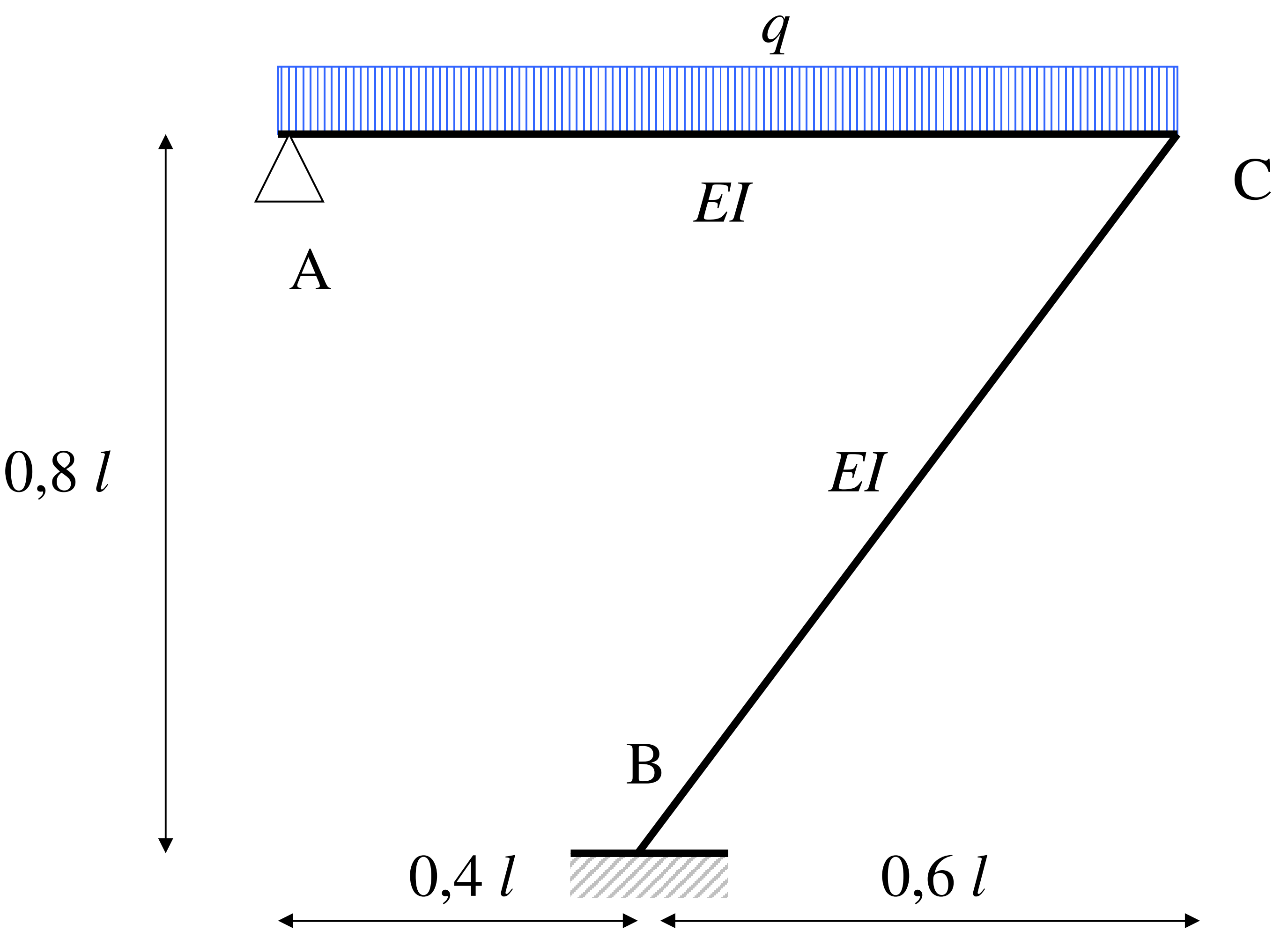
$$M_C=-37.6\ \text{kNm}$$

M-line

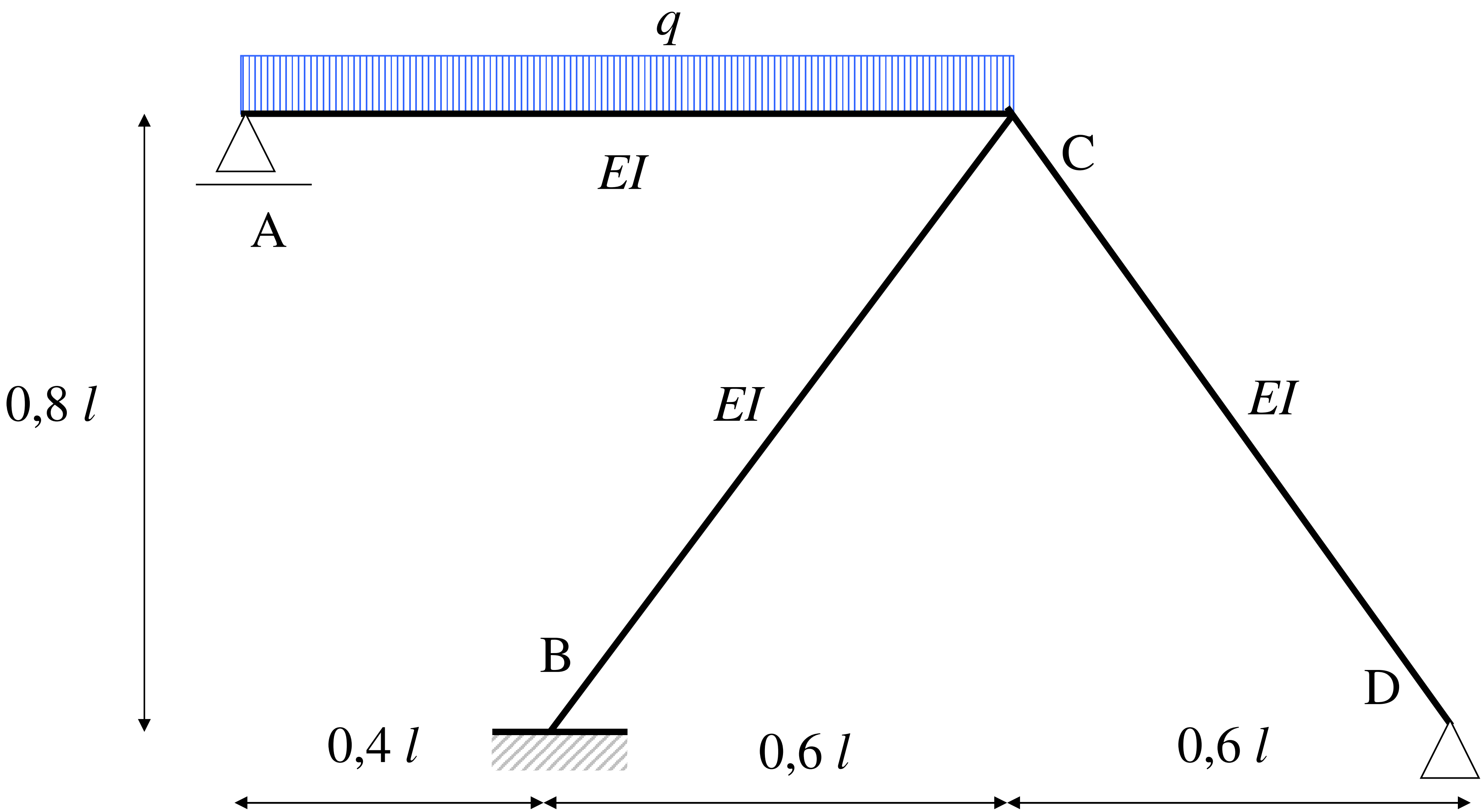


- Shear force distribution (V-line)
- Normal force distribution (N-line)
- Support reactions

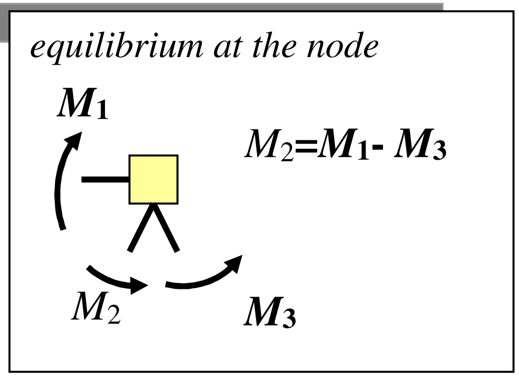
EXAMPLE 3 : FRAME



EXAMPLE 4 : FRAME



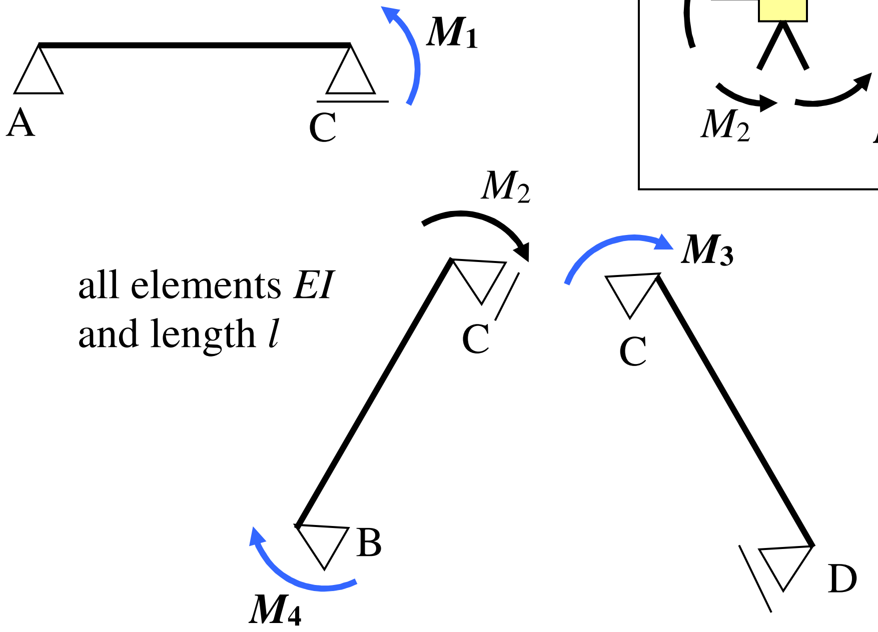
SOLUTION



$$\varphi_C^{(AC)} = \varphi_C^{(BC)}$$

$$\varphi_C^{(AC)} = \varphi_C^{(CD)}$$

$$\varphi_B = 0$$



$$\frac{M_1 l}{3EI} + \frac{ql^3}{24EI} = \frac{M_4 l}{6EI} - \frac{M_2 l}{3EI}$$

$$\frac{M_1 l}{3EI} + \frac{ql^3}{24EI} = -\frac{M_3 l}{3EI}$$

$$-\frac{M_4 l}{3EI} + \frac{M_2 l}{6EI} = 0 \quad \Rightarrow \quad M_4 = \tfrac{1}{2} M_2$$

3-fold statically indeterminate

3 static indeterminates
3 deformation conditions

SOLVING THE UNKNOWNS

Simplify the expressions by substitution of:

$$M_2 = M_1 - M_3 \qquad and \qquad M_4 = \tfrac{1}{2} M_2 = \tfrac{1}{2}(M_1 - M_3)$$

Thus:

$$\tfrac{7}{12} M_1 - \tfrac{1}{4} M_3 = -\tfrac{1}{24} q l^2 \qquad \times 4$$

$$\tfrac{1}{3} M_1 + \tfrac{1}{3} M_3 = -\tfrac{1}{24} q l^2 \qquad \times 3$$

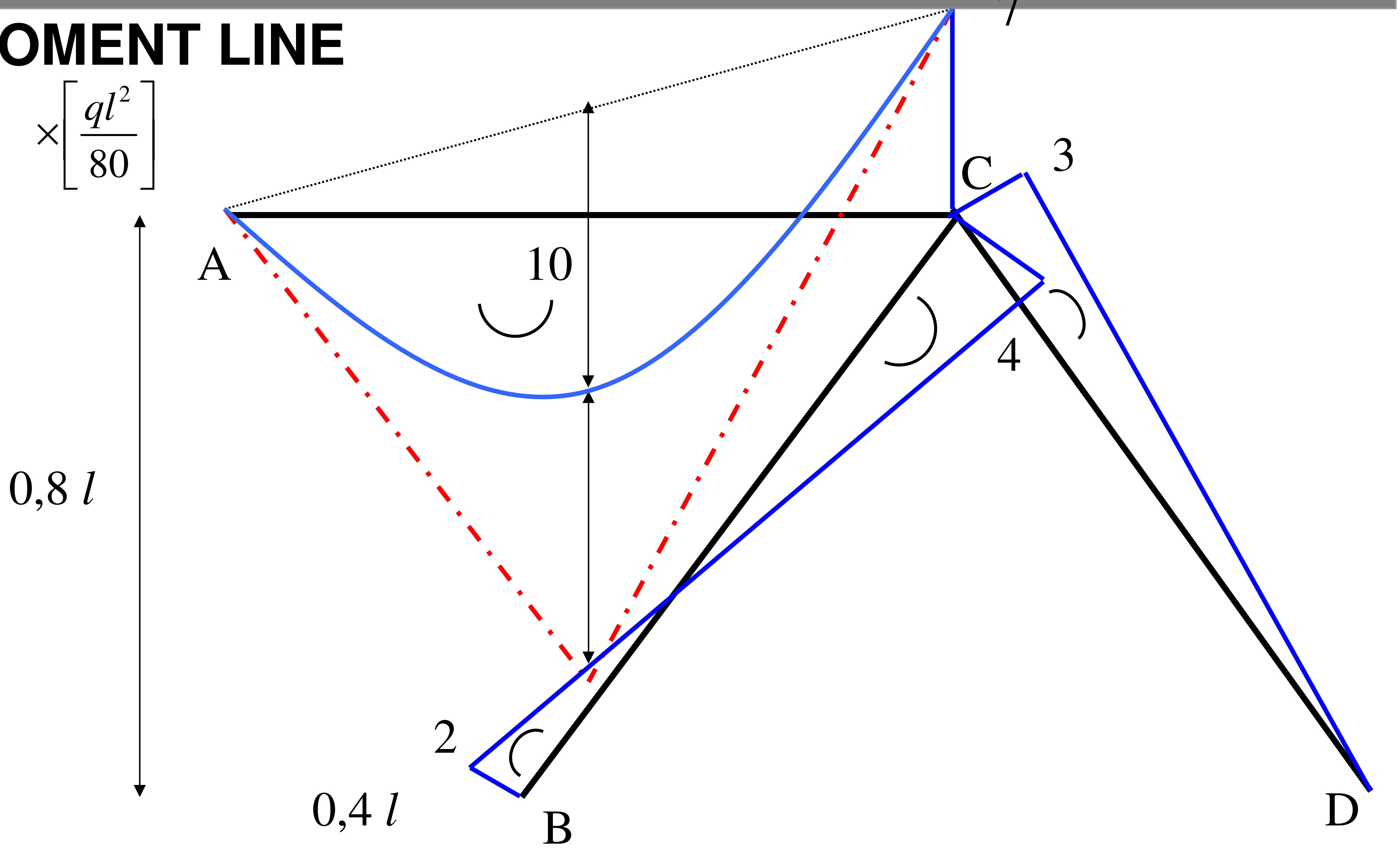
$+$

$$\tfrac{40}{12} M_1 = -\tfrac{7}{24} q l^2 \quad \Rightarrow \quad M_1 = -\tfrac{7}{80} q l^2 \qquad M_3 = -\tfrac{3}{80} q l^2$$

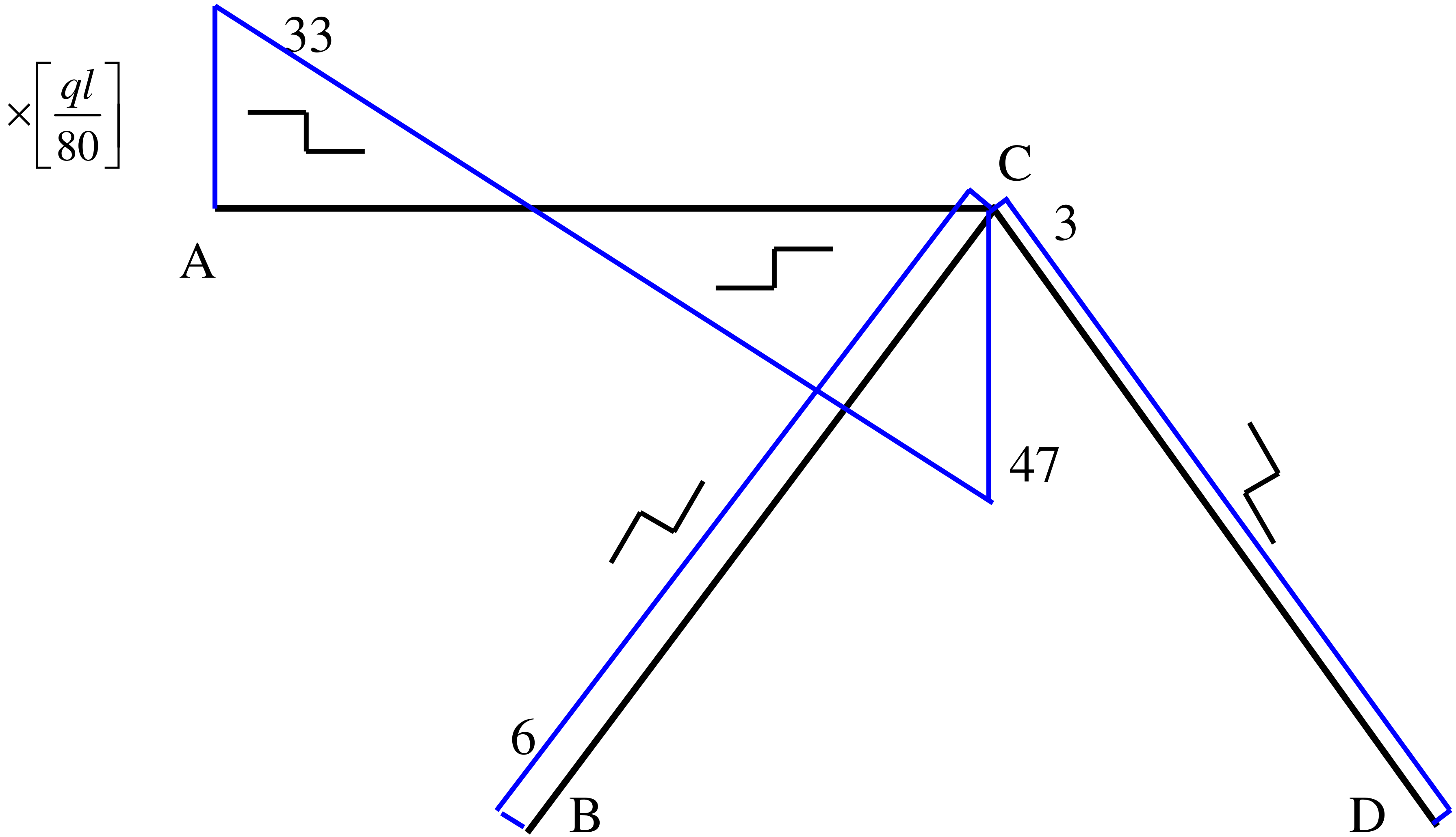
$$M_2 = -\tfrac{4}{80} q l^2 \qquad M_4 = -\tfrac{2}{80} q l^2$$

MOMENT LINE

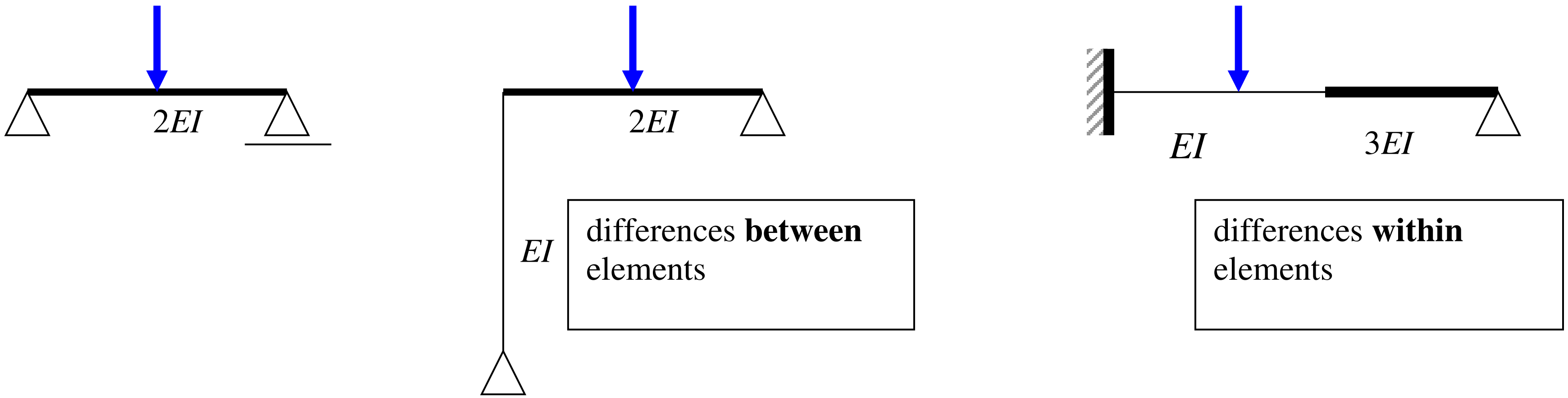
$$\times\left[\frac{ql^2}{80}\right]$$



SHEAR FORCE LINE



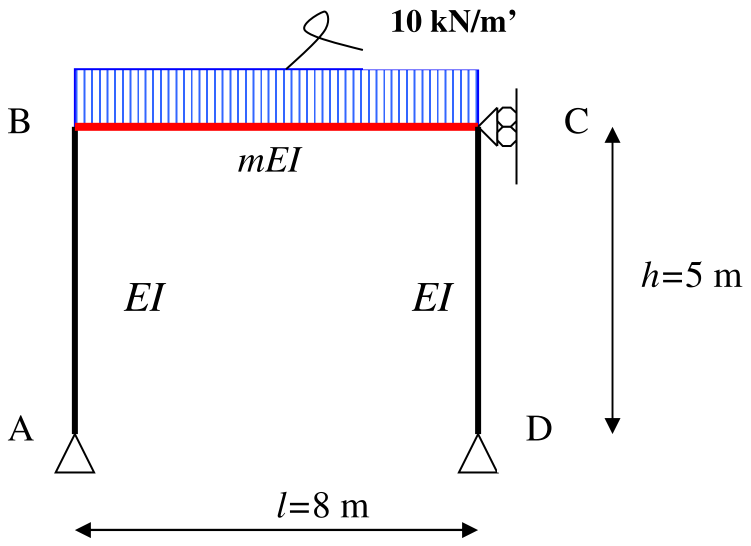
DIFFERENCES IN STIFFNESS WITHIN A STRUCTURE

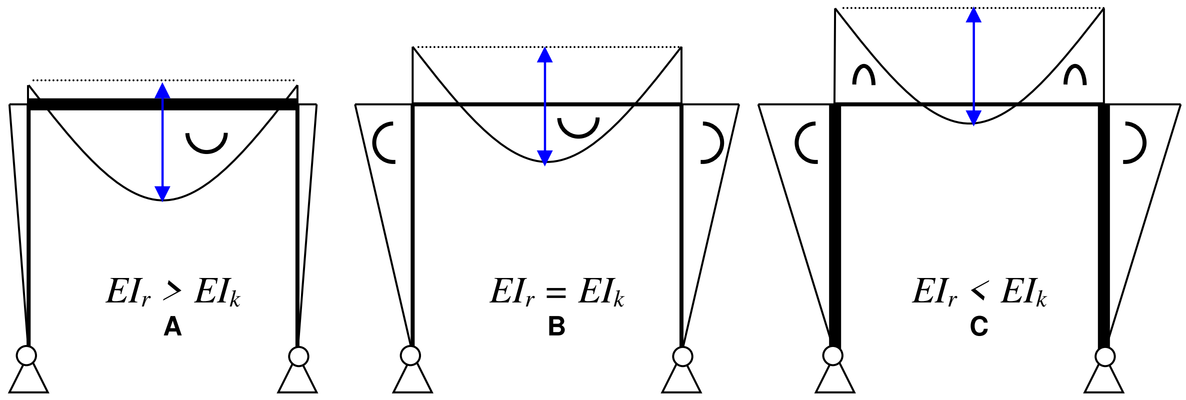


(a) *(b)* *(c)*

INFLUENCE ON THE FORCE DISTRIBUTION ?

EXAMPLE : SIMPLE FRAME





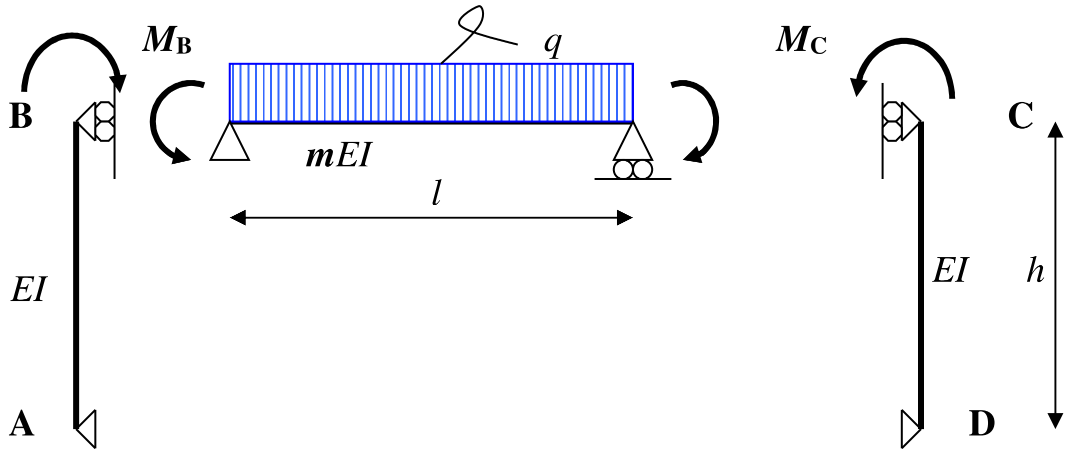
Moment distribution for three situations

$$m = \frac{EI_{beam}}{EI_{column}}$$

WHAT IS DE INFLUENCE OF m ON THE FORCE DISTRIBUTION ?

SOLUTION

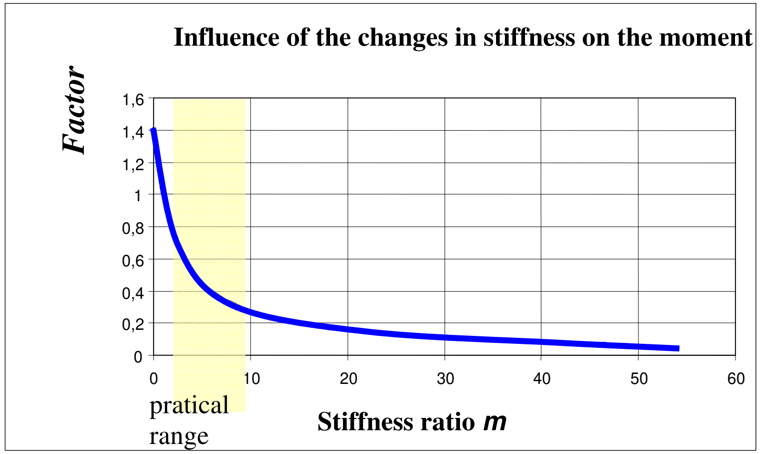
$$M_B = \frac{\frac{640}{m}\left(\frac{4}{m}+5\right)}{\frac{48}{m^2}+\frac{80}{m}+25}$$



FOR $m = 1$ => $M_B = 37{,}6$ kNm

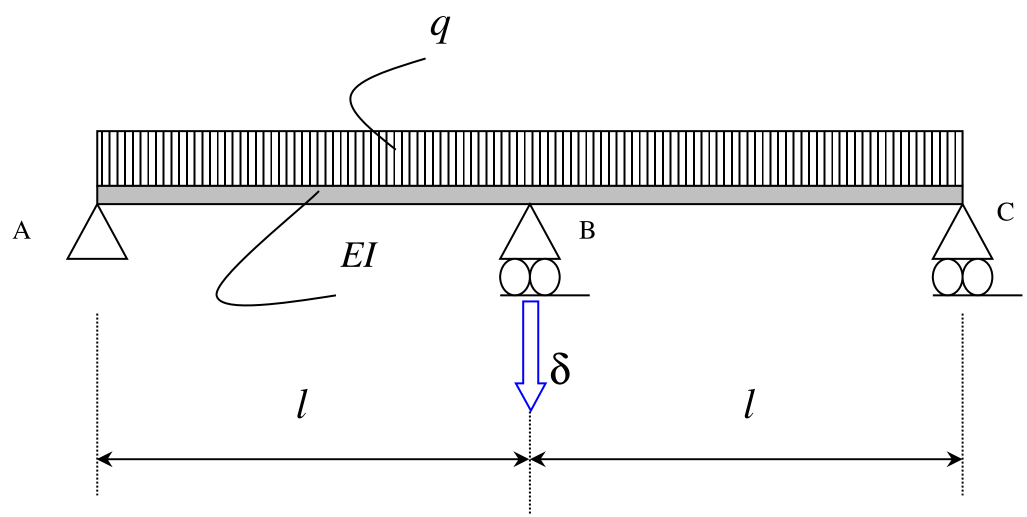
USE A FACTOR:

$$Factor = \frac{M_n}{M_1} = \frac{M_n}{37{,}6}$$

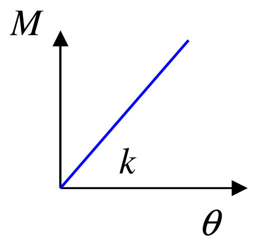


SPECIAL SITUATIONS

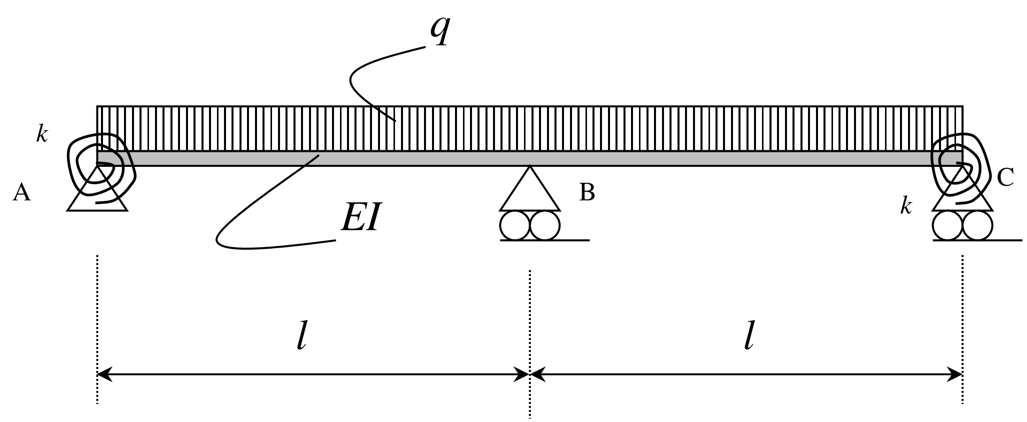
Settlement of the support



Flexible joints



LE rotational spring



SETTLEMENT OF A SUPPORT

APPROACH IS IDENTICAL AS WITH THE FORCE METHOD

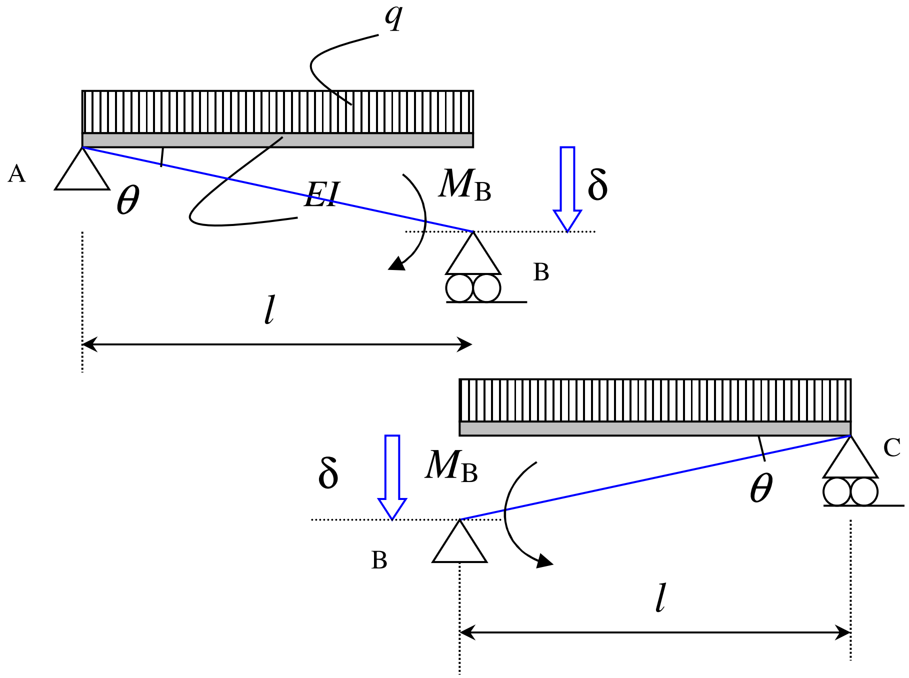
- choose a static determinate principal system
- set up the deformation conditions

ELABORATE THE DEFORMATION CONDITION

$$\frac{ql^3}{24EI} - \frac{M_B l}{3EI} - \theta = -\frac{ql^3}{24EI} + \frac{M_B l}{3EI} + \theta \quad \Leftrightarrow$$

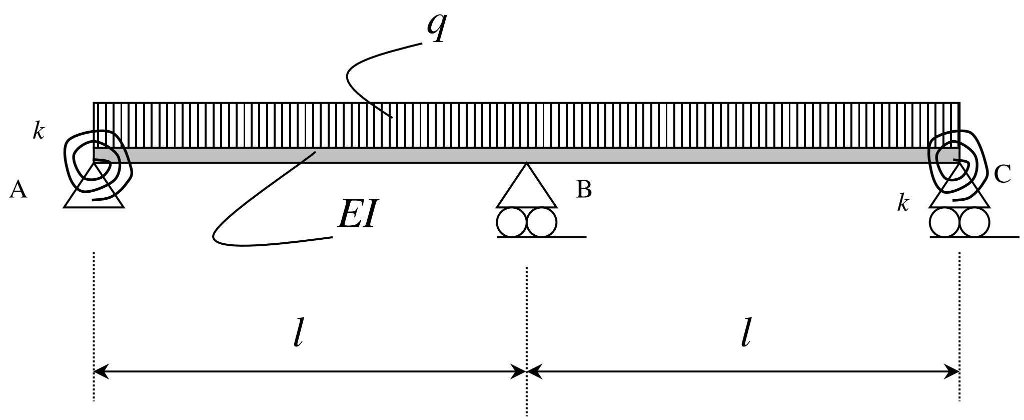
$$2M_B = \tfrac{1}{4}ql^2 - \frac{6EI}{l}\theta$$

$$M_B = \tfrac{1}{8}ql^2 - \frac{3EI}{l} \times \frac{\delta}{l} = \tfrac{1}{8}ql^2 - \frac{3EI}{l^2}\delta$$



FLEXIBLE JOINTS

SAME APPROACH



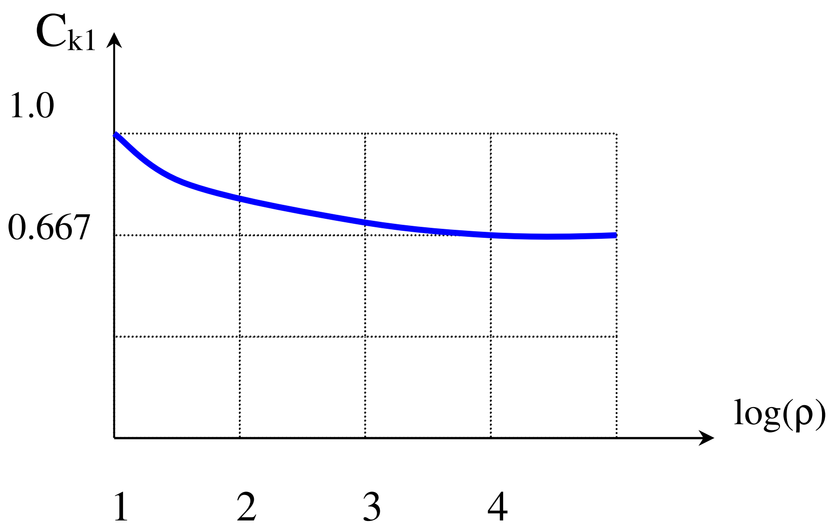
PROBLEM :

MOMENT IN THE SPRING DEPENDS ON THE YET UNKNOWN ROTATION OF THE BEAM END AT THE SUPPORT OR JOINT

SOLUTION FOR MOMENT AT B:

$$M_B = C_{k1} \tfrac{1}{8} q l^2$$

$$C_{k1} = \frac{1 + \frac{\rho}{6}}{1 + \frac{\rho}{4}} \qquad \rho = \frac{kl}{EI}$$



SOLUTION FOR MOMENT AT FLEXIBLE SUPPORT :

$$M_{spring} = C_{k2}\,\tfrac{1}{12}ql^2 \quad \text{with:} \quad C_{k2} = \frac{\frac{\rho}{4}}{1+\frac{\rho}{4}} \quad \text{and} \quad \rho = \frac{kl}{EI}$$

See for the complete derivation of the solution:

“NOTES ON THE FORCE METHOD”,
A brief note on support settlements moments in flexible joints and the effect of temperature loads on statically indeterminate continuous beams in bending, April 2013, J.W. Welleman.